Lecture 27

Time-Space Lower-bound for *SAT*

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Note: Don't confuse **TISP**$(T(n), S(n))$ with **DTIME**$(T(n)) \cap$ **DSPACE**$(S(n))$.

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Theorem: *SAT* $\notin$ TISP$(n^{1.1}, n^{0.1})$.

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Let $L \in$ NTIME(n) and A be a TISP$(n^{1.1}, n^{0.1})$ algorithm for *SAT*.

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(1) Generates required bits of ϕ_x from x in logspace, where $|\phi_x| = c|x|\log|x|$.

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Proof: *SAT* $\in$ TISP$(n^{1.1}, n^{0.1}) \Longrightarrow$ NTIME$(n) \subseteq$ TISP$(n^{1.2}, n^{0.2})$:

using tighter analysis
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$x \in L \iff$ $\exists C'$ s.t. $\exists$ a path of length at most $n^{12}/2$ from C_{start} to C' and from C' to C_{accept}

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(2) $\forall i$, $\exists$ a path of length at most from C_i to C_{i+1}.

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Σ_2TIME(n^8) algorithm for L on input x:

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$\Sigma_2\mathsf{TIME}(n^8)$ algorithm for L on input x:

(1) Generates $C_0, C_1, \ldots, C_{n^6}$ many configs. with $\exists$ state.

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Σ_2TIME(n^8) algorithm for L on input x:

(1) Generates $C_0, C_1, \ldots, C_{n^6}$ many configs. with ∃ state.

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(1) Generates $C_0, C_1, \ldots, C_{n^6}$ many configs. with $\exists$ state.

(2) Generates an $i \in [0, n^6 - 1]$ with $\forall$ state.

(3) Output $1 \iff C_0 = C_{start}$, $C_{n^6} = C_{accept}$ and C_{i+1} is reachable from C_i within n^6 steps.

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Σ_2TIME(n^8) algorithm for L on input x:

(1) Generates $C_0, C_1, \ldots, C_{n^6}$ many configs. with $\exists$ state.

Takes $O(n^8)$ time.

(2) Generates an $i \in [0, n^6 - 1]$ with $\forall$ state.

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(2) $\forall i$, $\exists$ a path of length at most n^6 from C_i to C_{i+1}.

$\Sigma_2\mathsf{TIME}(n^8)$ algorithm for L on input x:

(1) Generates $C_0, C_1, \ldots, C_{n^6}$ many configs. with $\exists$ state.

(2) Generates an $i \in [0, n^6 - 1]$ with $\forall$ state.

(3) Output $1 \iff C_0 = C_{start}$, $C_{n^6} = C_{accept}$ and C_{i+1} is reachable from C_i within n^6 steps.

Takes $O(n^8)$ time.

Time-Space Lower-Bound for *SAT*

Claim 1**:** $\mathsf{TISP}(n^{12}, n^2) \subseteq \Sigma_2\mathsf{TIME}(n^8)$.

Proof: Let $L \in \mathsf{TISP}(n^{12}, n^2)$ and M be a TM that decides L using n^{12} time and n^2 space.

Let $G_{M,x}$ denote the configuration graph of M on x. (include x in configs as well.)

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$$\text{NTIME}(n^{10}) \subseteq \text{TISP}(n^{12}, n^{2})$$

Time-Space Lower-Bound for *SAT*

Summary:

- We want to show $\mathit{SAT} \notin \text{TISP}(n^{1.1}, n^{0.1})$.
- Showing $\text{NTIME}(n) \not\subseteq \text{TISP}(n^{1.2}, n^{0.2})$ is sufficient.
- **Claim 1:** $\text{TISP}(n^{12}, n^2) \subseteq \Sigma_2\text{TIME}(n^8)$.
- **Claim 2:** If $\text{NTIME}(n) \subseteq \text{TISP}(n^{1.2}, n^{0.2})$, then $\Sigma_2\text{TIME}(n^8) \subseteq \text{NTIME}(n^{9.6})$.

Putting everything together:

Suppose $\text{NTIME}(n) \subseteq \text{TISP}(n^{1.2}, n^{0.2})$. Then,

$$\text{NTIME}(n^{10}) \underset{\substack{\uparrow \\ \text{from padding argument}}}{\subseteq} \text{TISP}(n^{12}, n^2)$$

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breaks nondeterministic time hierarchy theorem

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